Exploring Thermal Noise Limits to Spectral Hole Frequency Stability

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In the context of time and frequency metrology, ultra-frequency-stable lasers play a vital role as the tool for probing the atomic transitions in optical clocks. Frequency noise in the interrogating laser aliases to instability in the clock, and is thus a limiting technical noise source. In principle, frequency instability of the probe laser should be kept well below the fundamental stability limits of the optical clock if it is to achieve its maximum performance potential; this sets a performance target for ultra-stable lasers (a fractional frequency stability of order 10^{-18} over 1 s) to allow optical lattice clocks to reach their fundamental performance limit set by quantum projection noise.

Currently, state-of-the-art ultra-stable lasers utilize Fabry-Perot optical cavities as their frequency reference. This long established technique has been refined such that technical and environmental noise sources are reduced to below the fundamental physical noise floor, namely due to internal structural damping (i.e., Brownian thermal noise) at 1 s [1]. This has led to the availability of cavity-stabilized systems with instabilities of order 10^{-16} over 1 s.

A novel approach to frequency stabilization involves locking a laser to a narrow transmissive line in the homogenously broadened absorption spectrum of rare earth ions (Eu^{3+}) doped into a crystal host (Y_2SiO_5). The transmission line (or 'Spectral Hole') is formed by depleting the ions (or 'Burning') at the resonance frequency of a pump laser. First results of laser stabilization via Spectral Hole Burning (SHB) have produced instabilities down into the low-10^{-15} to high-10^{-16} range at 1 s [2][3]. The promising start has motivated the research into spectral-hole stability as a function of environmental parameters such as mechanical stress[4] and pressure[5], external electric fields[6], and temperature[*Under Review*]. Additionally, a novel multi-spectral-hole probing scheme has demonstrated greatly reduce detection noise from both shot noise as well as parasitic interference[7].

This still leaves the question of the fundamental physical limit to spectral hole stability. The work presented here addresses this point by deriving expressions for several sources of thermal noise. We exploit the fluctuation-dissipation theorem to calculate an expression for spectral-hole frequency instability due to internal mechanical dissipation (i.e., Brownian thermal noise). Additionally we calculate an expression for the instability resulting from thermal energy dissipation (i.e., thermodynamical thermo-spectral noise) as well as photon induced instability (i.e., photothermo-spectral noise). While, several necessary mechanical and thermal parameters of Y_2SiO_5 remain unknown at cryogenic temperatures, this work provides the basis for thermal-noise calculations as well as the direction for the future materials research needed to complete the estimates.

[1] M. Notcutt, L.-S. Ma, *et al.*, "Simple and compact 1-hz laser system via an improved mounting configuration of a reference cavity," *Opt. Lett.* **30**, 1815–1817 (2005).

[2] M. J. Thorpe, L. Rippe, *et al.*, "Frequency stabilization to 6×10^{-16} via spectral-hole burning," *Nature Photon.* pp. 688–693 (2011).

[3] N. Galland, N. Lučić, *et al.*, "Double-heterodyne probing for an ultra-stable laser based on spectral hole burning in a rare-earth-doped crystal," *Opt. Lett.* **45**, 1930–1933 (2020).

[4] N. Galland, N. Lučić, *et al.*, "Mechanical tunability of an ultranarrow spectral feature of a rare-earth-doped crystal via uniaxial stress," *Phys. Rev. Applied* **13**, 044022 (2020).

[5] S. Zhang, S. Seidelin, *et al.*, "First-order thermal insensitivity of the frequency of a narrow spectral hole in a crystal," *Phys. Rev. A* **107**, 013518 (2023).

[6] S. Zhang, N. Lučić, *et al.*, "Precision measurements of electric-field-induced frequency displacements of an ultranarrow optical transition in ions in a solid," *Applied Physics Letters* **117**, 221102 (2020).

[7] X. Lin, M. T. Hartman, *et al.*, "Multi-mode heterodyne laser interferometry realized via software defined radio," *Opt. Express* **31**, 38475–38493 (2023).